**Math 120  
3.7 Modeling Using Variation**

# **Objective:**

1. Solve direct variation problems.
2. Solve inverse variation problems.
3. Solve combined variation problems.
4. Solve problems involving joint and compound variation.

Here is an example showing the type of questions you can answer by writing and solving variation equations:

The force of wind blowing on a window positioned at a right angle to the direction of the wind varies jointly as the area of the window and the square of the wind’s speed. A 30-mph wind blowing on a 4 ft by 5ft window exerts a force of 150 lb. What force will be exerted on a 3ft by 4 ft window when a 60-mph wind blows on it? *(FYI, hurricane shutters need to be placed on any 12 ft2 window with an anticipated possible force of more than 300 lbs.)*

# **Topic #1: Direct Variation**

When two quantities vary **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**



one quantity is a constant multiple of the other. In direct variation, the quantities/variables move in the same direction. The most basic type of direct variation is:



This is read “ varies directly as ”. Both and are varying quantities and is the constant of variation/proportionality. With **direct variation**, *k* is being ***\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*** times *x*.



Suppose that varies directly as and that when . We can use the definition of direct variation to find the constant of variation : first, input the given values for *x* and *y*, then solve for *k*



Use the value found for k to write the direct variation **equation**:



This tells us that is always times the value of .

For example, if



*Example #1* – Find the Indicated Quantity

a) varies directly as and when . Find when .



Use the definition and given values for *x* and *y* to solve for k and write the general variation equation:



Evaluate at :



b) varies directly as and when . Find when .



Use the definition to solve for *k* and write the general variation equation



Evaluate at :



*Example #2* – Write the General Variation Equation that Models the Variation Described

a) varies directly as the square of .



The square of is expressed as . Using the definition of direct variation, write the general variation equation:



b) varies directly as the square root of .



The square root of is expressed as . Using the definition of direct variation, write the general variation equation:



c) varies directly as the cube of .



The cube of is expressed as . Using the definition of direct variation, write the general variation equation:



# **Topic #2: Inverse Variation**

Inverse variation is similar to direct variation; however, the quantities vary in opposite directions. The most basic type of inverse variation is:



This is read “ varies inversely as ”. Both and are varying quantities and is the constant of variation/proportionality.



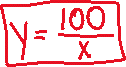
With **inverse variation**, the constant of proportionality, *k*, is being ***divided*** by *x*.



Suppose that varies inversely as and that when . We can use the definition of inverse variation to find the constant of variation :



Use the result to write the general variation equation:



This tells is that always equals 100. So, if , then

*Example #1* – Find the Indicated Quantity

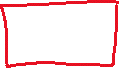
a) varies inversely as . When , . Find when .



Use the definition to solve for



Write the general variation equation:



Evaluate at



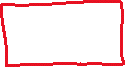
b) varies inversely as . When , . Find when .



Use the definition to solve for



Write the general variation equation:



Evaluate at



*Example #2* – Write the General Variation Equation that Models the Variation Described

a) varies inversely as the square of .



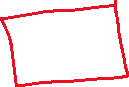
The square of is expressed as . Using the definition of inverse variation, write the general variation equation:



b) varies inversely as the square root of .



The square root of is expressed as . Using the definition of inverse variation, write the general variation equation:



# ***Topic #3: Joint Variation and Compound Variation***

***Joint Variation:***

**Joint variation** involves **three or more quantities that vary** **directly**. The most basic type of joint variation is:



This is read “ varies jointly as and ”. Where are varying quantities and is the constant of variation; with **joint variation**, k, x and z are all being ***multiplied***.



Suppose that varies jointly as and . When , and . We can use the definition of joint variation to find the constant of variation :



Using the value found for k, write the general variation equation:



***Compound Variation:***

Suppose we solve the equation above for



This tells us that varies **directly** as and **inversely** as . The constant of variation changed to .

This is an example of COMPOUND VARIATION. The most basic type of compound variation is:



This is read “ varies directly as and inversely as ”, where are varying quantities and is the constant of variation. ***k*** is ***multiplied*** time the variable that varies **directly**, and this product in the numerator is ***divided*** by the variable that varies **inversely**.



Suppose varies directly as and inversely as . When . We can use the definition of compound variation to solve for



Using the value found for k, write the general variation equation:



Note: We can solve any variation problem for any of the variables to come up with a different way to describe the relationship!

*Example #1* – Write the Equation that Models the Variation Described

a) varies jointly as and the square of .



The variables are . Use the definition of joint variation, write the general variation equation:



b) varies jointly as and the square root of and inversely as the square of .



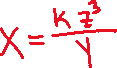
The variables are . This a compound situation involving JOINT and INVERSE, write the general variation equation:



c) varies directly as the cube of and inversely as .



The variables are . This is compound situation involving DIRECT and INVERSE, write the general variation equation:



# **Topic #4: Applications of Variation**

Many quantities are related through variation models.

*Example #1* – Analyze the Variation Model

a) The volume of blood, , in a person’s body varies directly as body weight, . A person who weighs 160 pounds has about 5 quarts of blood. Estimate the volume of blood in a person who weighs 200 pounds.



Let B be:



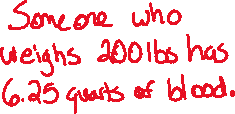
Let W be:



Use the given quantities to solve for



Evaluate the model when



b) The distance, , that a body falls from rest varies directly as the square of time, , of the fall. If a skydiver falls 64 feet in 2 seconds, how far will they fall in 4.5 seconds?



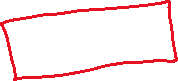
Let *s* be:



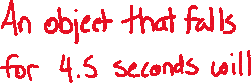
Let *t* be:



Use the given quantities to solve for



Evaluate the model when



c) A radiation machine produces an intensity of radiation that varies inversely as the square of the distance from the machine. At 3 meters, the radiation intensity is 62.5 milliroentgens per hour. What is the intensity at a distance of 2.5 meters?



The variables were not assigned; choose any desired variable for the quantities. For example, intensity could be and distance could be .

Let be:



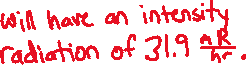
Let be:



Use the given quantities to solve for



Evaluate the model when . Round to the nearest tenth.



d) Kinetic energy varies jointly as the mass and the square of the velocity. A mass of 8 grams and a velocity of 3 centimeters per second has a kinetic energy of 36 ergs. Find the kinetic energy for a mass of 4 grams and velocity of 6 centimeters per second.



The variables were not assigned; choose any desired variable for the quantities. For example, kinetic energy could be , mass could be , and velocity could be .

Translate the joint variation model into a general variation equation:

Let be:



Let be:



Let be:



Use the given quantities to solve for



Evaluate the model when



e) The body-mass index (BMI) varies directly as one’s weight, in pounds, and inversely as the square of one’s height, in inches. A BMI between 20 and 25 (inclusive) is considered normal. Values out of this range are considered underweight when below 20 and overweight when above 25. Suppose that a person who weighs 180 pounds and is 60 inches tall (5 feet) has a BMI of 35.15. Using this information, what is the BMI of a person who weighs 170 pounds and is 5 feet 10 inches tall? Round to the nearest tenth. Is this person overweight? Is the person underweight?



The variables were not assigned; choose any desired variable for the quantities. For example, BMI could be , weight could be , and height could be .

Let be:



Let be:



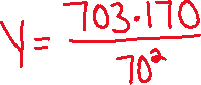
Let be:



Use the given quantities to solve for



Evaluate the model when (this is the height in inches, recall 12 inches = 1 foot):



What is this person’s BMI? Is this person overweight? Underweight?

